Aug 30, 2023 versio

Calc AB Test Practice Ch P,1

Name: Ɗ Chris Block: Seat:

Calculator skills to know:

1..... How to make sure you are in RADIAN mode.

2..... How to graph two functions to find their intersections.

3..... How to graph a functions to find its zeros, end behavior, and asymptotes.

4..... How to Use the TBL SET and TABLE feature to quickly evaluate a function for values around a point

5..... How to Use the WINDOW and ZOOM feature to quickly TRACE a function for values around a point

6..... How to type functions that have absolute value, or greatest integer functions (MATH NUM 1:abs, 5:int)

Calculus skills to know:

1..... How to find a one or two sided limit from a graph.

2..... How to find a one or two sided limit from a table.

3..... Understand what a limit is, and whether they exist, or are infinite, or do not exist.

4..... How to find a one or two sided limit from a formula, using clever algebra tricks, the properties of limits, and a few theorems (like Thm 1.1-1.6, 1.9, 1.15).

5..... How to read and write a limit in proper mathematical notation.

6..... Knows what Continuity is, and how to justify whether a function is continuous or not (at a point, an interval, or everywhere) from a Thm (1.11, 1.12) or the definition (p 74, 77)

7..... How to find a value to make a piece-wise function continuous

8..... How to use the IVT (Thm 1.13) to justify a zero (or any other value) on a closed interval

Precalculus skills to know:

1...... How to find the equation of a line from 2 points or with slope and one point

2...... Know how to the equation of vertical and horizontal lines.

3...... Graph a piece-wise function

4...... Understand function notation (like $f(x) = 5$)

5...... Understand interval notation (like $(-\infty, 4]$)

6...... Finding the domain of functions (check for division by zero or the square root of a negative)

7...... How to factor $(a^2 - b^2)$, $(a^3 \pm b^2)$, and expand $(a \pm b)^2$

8...... How to factor polynomials & rational functions to discover the zeros, asymptotes, and holes (discontinuities)

Remote Testing

1. Have a device that runs Zoom, and another to take the test with. These are not ideal conditions, so we need to maintain audio/visual contact throughout the test to deal with glitches as they arise.
2. You are permitted 60 minutes (90 min for those who qualified for extra time). Some questions are "no calculator permitted" and some are "calculator active." (2:1 ratio on the AP Test)
3. All questions will be online at TestPortal.net. A link will be emailed to you at the start of the test.
4. Most questions can be answered on TestPortal (multiple choice, or a short answer question). A few questions (ones like 3 5, 6, 8, 13, 17/18) are "Free Response" and need to be answered by sketching a graph or doing some algebraic steps on paper or an iPad. This is the way you do your homework, so I hope you are able to upload them to google classroom the usual way (by photographing your paper, or submitting a pdf).
5. For credit on these free response questions, you must upload then by the end of the test. You will have a grace period of 3 minutes, and you may only submit it once. If you upload anything later, or remove or replace something you uploaded, you will score 0 points on those questions.
6. Any make-up Tests need to be taken before the next class.

"No Calculator" Practice

Be neat.
For full credit show all work in an orderly way, as if to express your reasoning to another person.

1. (6 points) Given $f(x) = \begin{cases} -2x^2 + 4x + 1, & x \geq 1 \\ 4 - x, & x < 1 \end{cases}$

   Find:
   (show the piece of the function you are using)

   (a)
   $$\lim_{x \to 1^+} f(x)$$

   from right
   $= \lim_{x \to 1^+} -2x^2+4x+1$
   $= -2(1)^2+4(1)+1 = 3$

   (b)
   $$\lim_{x \to 1^-} f(x)$$

   from left
   $\lim_{x \to 1^-} 4 - x$
   $4-(1)$
   3

   (c)
   $$\lim_{x \to 1} f(x)$$

   Since
   $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = 3$
   $\lim_{x \to 1} f(x) = 3$

2. (10 points) Evaluate the following limits. Show the work that leads to your answer.

   (a)
   $$\lim_{x \to 4} \frac{(x-4)^2}{x^2-16}$$

   ($\frac{0}{0}$ indet. form)

   $\lim_{x \to 4} \frac{\cancel{(x-4)}(x-4)}{\cancel{(x-4)}(x+4)}$
   $= \lim_{x \to 4} \frac{x-4}{x+4} = \frac{4-4}{4+4} = 0$

   (b)
   $$\lim_{x \to \pi} \frac{\sec x}{x}$$

   (type 1)

   $= \lim_{x \to \pi} \frac{1}{x \cos x} = \frac{1}{\pi(-1)} = \frac{-1}{\pi}$

   (c)
   $$\lim_{x \to \frac{\pi}{2}} \frac{\csc x}{x}$$

   (Type 1)

   $= \lim_{x \to \frac{\pi}{2}} \frac{1}{x \sin x} = \frac{1}{\frac{\pi}{2}(1)} = \frac{2}{\pi}$

   (d)
   $$\lim_{x \to 0} \frac{3x}{\sin 3x}$$

   Since by Thm 1.9 $\lim_{x \to 0} \frac{\sin x}{x} = 1 = \lim_{x \to} \frac{x}{\sin x}$
   $\lim_{x \to 0} \frac{3x}{\sin 3x} = 1$

   (e)
   $$\lim_{x \to 1} \frac{x^3-1}{x^2+x+1}$$

   $\frac{0}{\text{number}}$ - Type 1

   $\lim_{x \to 1} \frac{(x-1)(x^2+x+1)}{(x^2+x+1)} = 0$

3. (12 points) Evaluate the following limits. Show the work that leads to your answer.

(a)
$$\lim_{x \to 0^-} \frac{\cos x}{x}$$

(Type 2)

$$\lim_{x \to 0^-} \frac{\cos 0}{0}$$

$$\lim_{x \to 0^-} \frac{1}{x} = -\infty$$

(b)
$$\lim_{x \to -2} \frac{x^2 - x - 6}{x + 2}$$

"Type 3" ($\frac{0}{0}$ ind. form)

$$\lim_{x \to -2} \frac{\cancel{(x+2)}(x-3)}{\cancel{x+2}}$$

$$\lim_{x \to -2} x - 3 = -5$$

(c)
$$\lim_{x \to 0} \frac{\sqrt{1 + 2x} - 1}{x}$$

"Type 3" $\frac{0}{0}$ form (indeterm.)

$$\lim_{x \to 0} \frac{(\sqrt{1+2x} - 1)(\sqrt{1+2x} + 1)}{x\,(\sqrt{1+2x} + 1)}$$

$$\lim_{x \to 0} \frac{(\cancel{1} + 2\cancel{x}) - \cancel{1}}{\cancel{x}(\sqrt{1+2x} + 1)} = \frac{2}{\sqrt{1+2(0)} + 1} = \frac{2}{2} = 1$$

(d)
$$\lim_{x \to 0} \frac{\tan^2 x}{x}$$

"Type 3" ($\frac{0}{0}$)

$$\lim_{x \to 0} \left(\overset{1}{\cancel{\frac{\sin x}{x}}}\right) \cdot \frac{\sin x}{\cos^2 x}$$

$$\lim_{x \to 0} \frac{\tan x}{\cos x} = \frac{0}{1} = 0$$

4. (3 points) Given $f(x) = \begin{cases} -2x^2 + 4x + 1, & x \geq 1 \\ 4 - x, & x < 1 \end{cases}$

repeat of 1

(a) $\lim_{x \to 1^-} f(x) =$

from left

$$= \lim_{x \to 1^-} 4 - x = 4 - 1 = 3$$

(b) $\lim_{x \to 1^+} f(x) =$

from right

$$= \lim_{x \to 1^+} -2x^2 + 4x + 1$$

$$= -2(1)^2 + 4(1) + 1 = -2 + 4 + 1 = 3$$

(c) $\lim_{x \to 1} f(x) =$

Since $\lim_{x \to 1^-} f(x) = 3$

and $\lim_{x \to 1^+} f(x) = 3$

$$\lim_{x \to 1} f(x) = 3$$

5. (2 points) $\lim_{x \to 1} \frac{x^2 - 1}{x^2 + 2x - 3}$

($\frac{0}{0}$ indet. form) "Type 3"

$$= \lim_{x \to 1} \frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}(x+3)}$$

$$= \lim_{x \to 1} \frac{x+1}{x+3} = \frac{2}{4} = \frac{1}{2}$$

6. (3 points)
$$\lim_{x \to 0} \frac{\sin 5x}{2x}$$

($\frac{0}{0}$ indet. form) "Type 3"

$$\lim_{x \to 0} \left(\overset{1}{\cancel{\frac{\sin 5x}{5x}}}\right) \cdot \frac{5x}{2x}$$

$$= \lim_{x \to} \frac{5\cancel{x}}{2\cancel{x}} = \frac{5}{2}$$

Recall "point slope" form
$(y - y_0) = m(x - x_0)$

7. (4 points) Write an equation of the linear function f where $f(-1) = 5$ and $f(3) = -7$

$m = \frac{\Delta y}{\Delta x} = \frac{-7-5}{3-(-1)} = \frac{-12}{4} = -3$

$y - 5 = -3(x+1)$

8. (6 points) Write the function $f(x) = |x-3| + 2x$ as a piece-wise function and sketch its graph. Be sure to include the point where the slope changes.

neg slope before 3
pos slope after 3

$f(x) = \begin{cases} -(x-3)+2x, & x \le 3 \\ (x-3)+2x, & x > 3 \end{cases}$

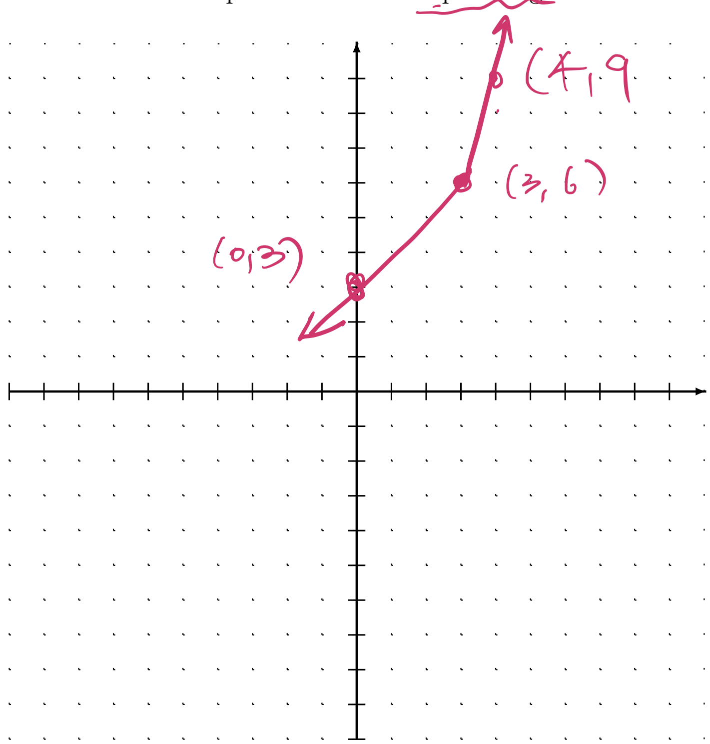
or

$f(x) = \begin{cases} x+3, & x \le 3 & \text{(slope 1)} \\ 3x-3, & x > 3 & \text{(slope 3)} \end{cases}$

or

$f(x) = \begin{cases} x+3, & x < 3 \\ 3(x-1), & x \ge 3 \end{cases}$

etc...



x	0	3	4
$f(x)$	$0+3$ 3	$3+3$ 6	$3(4-1)$ 9

9. (2 points) Find the domain of $g(x) = \sqrt{9-2x}$. Show all work.

$9 - 2x \ge 0$

$9 \ge 2x$

$2x \le 9$

$x \le \frac{9}{2}$

Domain is $(-\infty, \frac{9}{2}]$

10. Let $f(x) = x^3 - 2x^2 - 15x$

(a) (3 points) Find all the zeros of $f(x)$

$x(x^2 - 2x - 15) = 0$

$x(x+3)(x-5) = 0$

$\{0, -3, 5\}$

(b) (4 points) Using $f(x)$ (defined above), let

$$h(x) = \begin{cases} \dfrac{f(x)}{x-5}, & x \neq 5 \\ k, & x = 5 \end{cases}$$

Find the value of k so that $h(x)$ is continuous at $x = 5$. Justify your answer.

$$h(x) = \begin{cases} \dfrac{x(x+3)\cancel{(x-5)}}{\cancel{(x-5)}}, & x \neq 5 \\ k, & x = 5 \end{cases}$$

$\lim_{x \to 5} h(x) = \lim_{x \to 5} x(x+3) = 5(5+3) = 40$

so $k = 40$ so that

$\lim_{x \to 5} h(x) = h(x)$; Req by Def. of Continuity

11. (3 points) Explain why the function

$$f(x) = 1 - 3x - x^3$$

has a zero on the closed interval $[0, 1]$ – use IVT!

1. Since f is a polynomial, f is cont.
2. Since cont., IVT applies
3. Since $f(1) = -3$ and $f(0) = 1$

and $-3 < 0 < 1$

by IVT, there exists a c where $0 \leq c \leq 1$ such that $f(c) = 0$.

12. Let $f(x) = \dfrac{x+1}{x^2-3x-4}$.

(a) (2 points) For what value(s) of x does $f(x)$ have a discontinuity?

Since $f(x) = \dfrac{(x+1)}{(x+1)(x-4)}$

when $x=-1$ or $x=4$

(b) (2 points) Write equations(s) of any vertical asymptotes

$x=4$ is the vertical asymptote

(c) (4 points) At each point of discontinuity found in part (a), determine whether or not the limit exists. Justify your answer.

$x=-1$ is a removable discont.

$\lim_{x\to -1} f(x) = -\frac{1}{5}$

$x=4$

$\lim_{x\to 4^-} f(x) = -\infty$, $\lim_{x\to 4^+} f(x) = +\infty$ } so $\lim_{x\to 4}$ DNE.

(d) (2 points) A rational function $g(x) = \dfrac{a}{x+b}$ is defined such that $g(x) = f(x)$ whenever $f(x)$ is defined. Using the function $f(x)$ from part (a), determine the values of a and b.

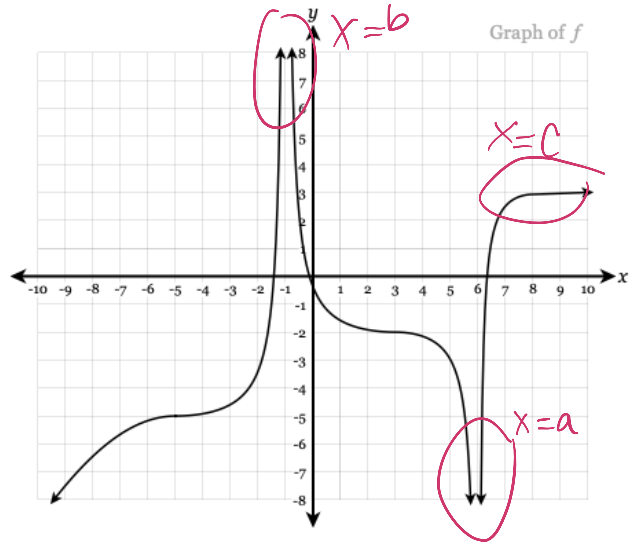
$\lim_{x\to -1} f(x) = \dfrac{(\cancel{x+1})}{(x-4)(\cancel{x+1})} = \dfrac{1}{x-4}$

$g(x) = \dfrac{a}{x+b} = \dfrac{1}{x-4}$

so $a=1$

$b=-4$

13. (3 points)



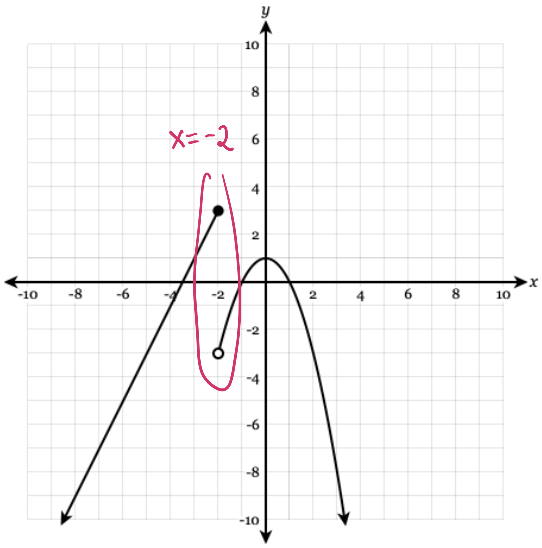
From the graph of f above, find the value.

(a) If $\lim_{x\to a} f(x) = -\infty$, $a =$ 6

(b) If $\lim_{x\to b} f(x) = \infty$, $b =$ 1

(c) If $\lim_{x\to \infty} f(x) = c$, $c =$ 3

14. (6 points) Consider the graph of f below.



Based on this graph, for what values of x is the function f not continuous? Justify your conclusion with mathematical reasoning using the values given in the graph.

$\lim_{x \to -2^-} f(x) = 3$

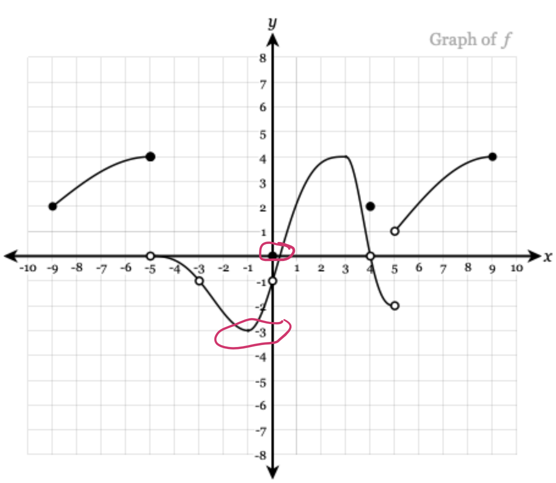
$\lim_{x \to -2^+} f(x) = -3$

$\lim_{x \to -2^-} f(x) \neq \lim_{x \to -2^+} f(x)$ so $\lim_{x \to -2} f(x)$ DNE, Hence not cont. at $x = -2$

Recall Def. of Cont:

1. $\lim_{x \to a} f(x)$ exists
2. $f(a)$ exists
3. $f(a) = \lim_{x \to a} f(x)$ (1 & 2 matches)

15. (5 points) Consider the graph of $f(x)$ below. and find the missing value indicated by the question mark (?).



$\lim_{x \to ?} f(x) = -3$? = -1

$f(?) = 0$? = 0 (and around 0.2)

$\lim_{x \to -5^-} f(x) = ?$? = 4

$f(4) = ?$? = 2

$\lim_{x \to 4} f(\cancel{4})$ f(x) (typo) $= ?$? = 0

Part 2 "Calculators Active" Practice (Though they are not always necessary)

16. (5 points) Graph a function f that has the following criteria: Many solutions, here is one ....

try one and check to make sure you have each!

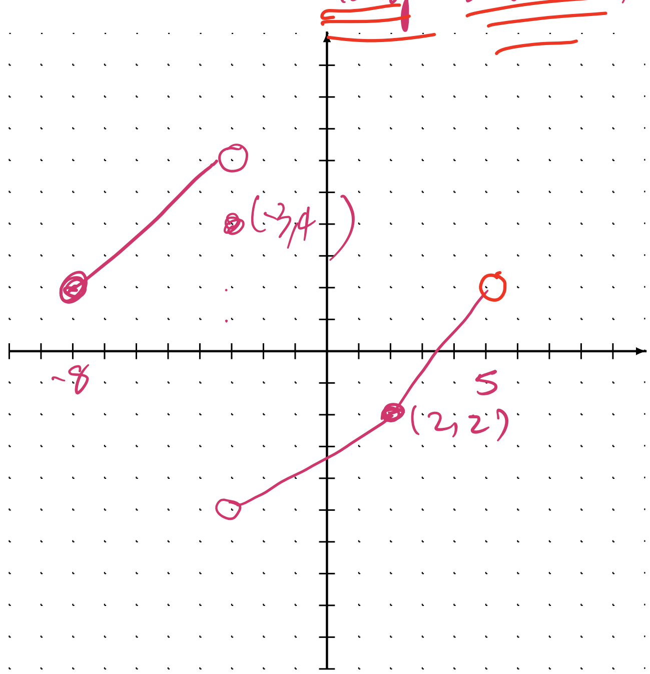
(a) Domain of $[-8, 5)$

(b) $\lim_{x \to 2} f(x) = -2$

(c) $f(-3) = 4$

(d) $\lim_{x \to -3^-} f(x) = 6$

(e) $\lim_{x \to -3^+} f(x) = -4$



17. (2 points) Using the graph of f below, choose true or false:

(a) $\lim_{x \to -2} f(x) = -2$

true

(b) f is continuous on $(-5, -2)$ ← $(-5 < x < -2)$, an open interval

true

(c) $f(1) = -3$

true

(d) $\lim_{x \to -5^+} f(x) = \cancel{1}\ 3$

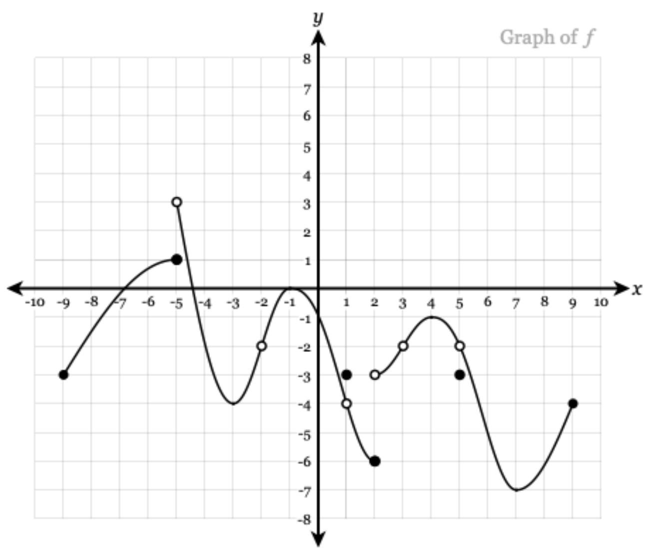
false

(e) f has a removable discontinuity at $x = 3$ true

$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x)$

(f) $\lim_{x \to 2^+} f(x)$ does not exist

false

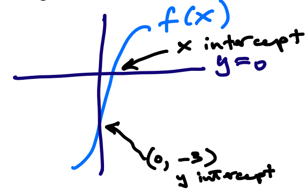


$\lim_{x \to 2^+} f(x) = -3$

but $\lim_{x \to 2} f(x)$ DNE since $-6 = \lim_{x \to 2^-} f(x) \neq \lim_{x \to 2^+} f(x) = -3$

Give final answers to three decimal places, either rounded or truncated

18. (2 points) Find the x intercept(s) of the function $f(x) = 3x^3 - 4x^2 + 3x - 3$



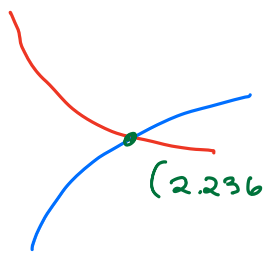
$f(1.1962111) = 0$

(1.196 is fine)

“3 for AP”

19. (3 points) Find the the point(s) (x, y) of intersection of the functions f and g:

$$f(x) = \ln(x - 1)$$
$$g(x) = 2^{-x}$$



(2.236, 0.212)

“3 for AP” - rounded or truncated

20. (2 points) Let $f(x) = \dfrac{\cos x}{x - \pi}$. Use the numerical method to evaluate

$$\lim_{x \to \pi^-} f(x) = +\infty$$

(note that this is a onesided limit, so use numbers less than π)

x	3.0	3.1	3.14	3.141	3.14159
$f(x)$	6.992	24.022	627.89	1,687.3	376,848

21. (2 points) Let $f(x) = \dfrac{\cos x}{x - \pi}$. Use the numerical method to evaluate

$$\lim_{x \to \pi} f(x) = \text{DNE}$$

Since $\lim_{x \to \pi^-} f(x) = +\infty$ and $\lim_{x \to \pi^+} f(x) = -\infty$

x	3.1	3.14	π	3.142	3.15
$f(x)$	24.022	627	undef	-2455	-118.9

(note that this is a two sided limit, so use numbers smaller <u>AND</u> larger than π)